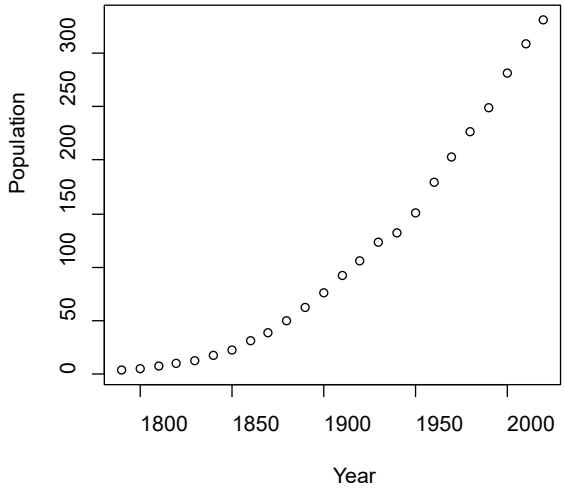
 # Dr. M. Baron, Statistical Machine Learning class, STAT-427/627

Polynomial Regression

 # Example: QUADRATIC MODEL FOR PREDICTING THE US POPULATION

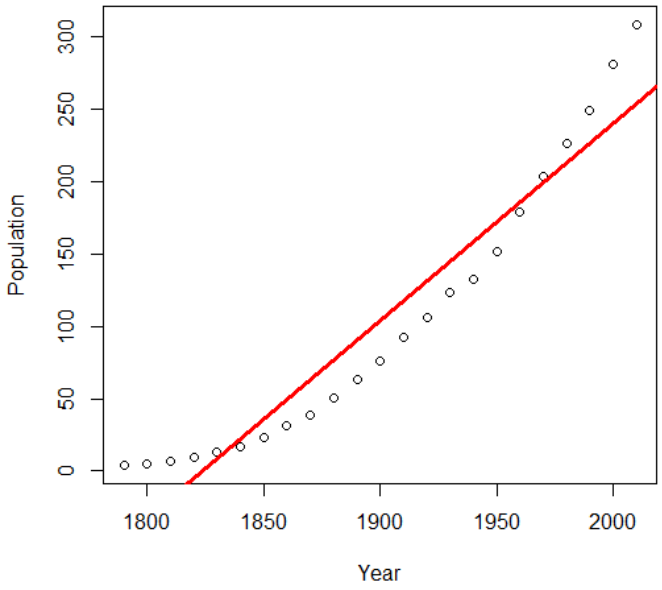
```
> setwd("C:\\Users\\baron\\Documents\\Teach\\627 Statistical Machine
Learning\\Data")

> Data = read.csv("USpop.csv")
> names(Data)
[1] "Year"       "Population"

> attach(Data)
> plot(Year,Population)
```



 # LINEAR MODEL

```
> lin = lm(Population ~ Year)
> summary(lin)

Call:
lm(formula = Population ~ Year)

Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept) -2.600e+03  1.691e+02  -15.38 2.98e-13 ***
Year         1.425e+00  8.871e-02   16.06 1.24e-13 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 30.08 on 22 degrees of freedom
Multiple R-squared:  0.9214,    Adjusted R-squared:  0.9178
F-statistic: 257.9 on 1 and 22 DF,  p-value: 1.235e-13
```

```
> abline(lin,col="red",lwd=3)
```

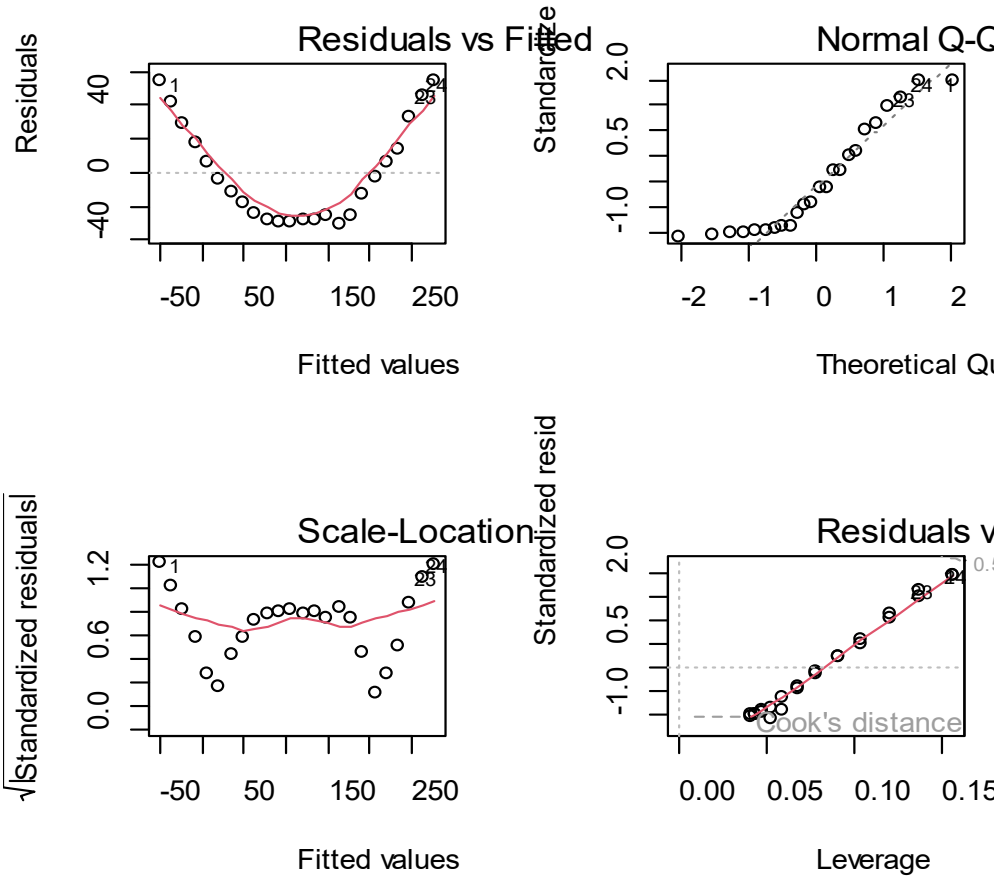


 # Clearly, the linear model is too inflexible and restrictive, it does not provide a good fit.
 # This is underfitting. Notice, however, that R2 in this regression is 0.9193. Without looking
 # at the plot, we could have assumed that the model is very good!

```
> predict(lin,data.frame(Year=2030))
       1
291.5174
```

 # This is obviously a poor prediction. The US population was already 331 million during the most
 # recent Census. So, we probably omitted an important predictor. Residual plots will help us
 # determine which one.
 # Let's produce various related plots. Partition the graphics window into 4 parts and use "plot".

```
> par(mfrow=c(2,2))
> plot(lin)
```



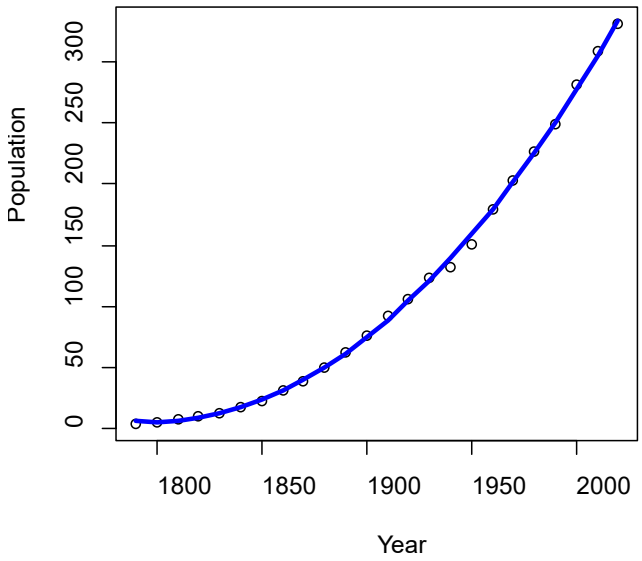
 # The first plot shows that a quadratic term has been omitted although
 # it is important in the population growth. So, fit a quadratic model.
 # Command I(…) means "inhibit interpretation", it forces R to understand (…)
 # literally, as Year squared.

```
> quadr = lm(Population ~ Year + I(Year^2))
> summary(quadr)

               Estimate Std. Error t value Pr(>|t|)
(Intercept)  2.170e+04  5.227e+02   41.52   <2e-16 ***
Year        -2.412e+01  5.493e-01  -43.91   <2e-16 ***
I(Year^2)    6.705e-03  1.442e-04   46.51   <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 3.019 on 21 degrees of freedom
Multiple R-squared:  0.9992,    Adjusted R-squared:  0.9992
F-statistic: 1.389e+04 on 2 and 21 DF,  p-value: < 2.2e-16
```

 # A higher R-squared is not surprising. It will always increase when we add
 # new variables to the model. The fair criterion is Adjusted R-squared, when
 # we compare models with a different number of parameters. Quadratic model has
 # Adjusted R-squared = 0.999 comparing with 0.9155 for the linear model.

 # Now let's obtain the fitted values and plot the fitted curve.

```
> Yhat = fitted.values(quadr)

> lines(Year,Yhat,col="blue",lwd=3)
```




```
> predict(quadr,data.frame(Year=2030))
       1
364.1572
```

 # Now, this is a reasonable prediction for the year 2030.

```
> predict(quadr,data.frame(Year=2030),interval="confidence")
       fit      lwr      upr
1 364.1572 359.9686 368.3459

> predict(quadr,data.frame(Year=2030),interval="prediction")
       fit      lwr      upr
1 364.1572 356.6102 371.7042
```

 # Food for thought... Are the confidence and predictions intervals valid here?